

Stress in Viscous Potential Flow

$$T_{ij} + p\delta_{ij} = \mu \left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] = 2\mu \frac{\partial^2 \phi}{\partial x_i \partial x_j}$$

- Viscosity may generate a significant contribution to the stress in regions of irrotational flow

Joseph, DD and Liao, T 1994. Potential flows of viscous and viscoelastic fluids, *J. Fluid Mech.* **265**, 1-23.

Joseph, DD and Liao, T 1994. Viscous and viscoelastic potential flow, *Trends and Perspectives in Applied Mathematics, Applied Mathematical Sciences*, Sirovich, Arnol'd, eds, Springer-Verlag. Also in Army HPCRC preprint 93-010, **100**, 1-54.

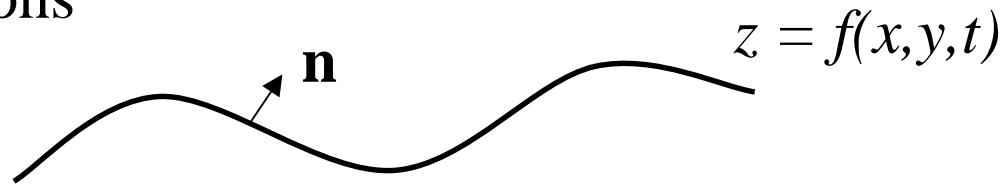
Viscous Potential Flow

- The theory of potential flow of an inviscid fluid can be replaced with the theory of potential flow of a viscous fluid. The drag on a body in potential flow is independent of the viscosity but there is an additional viscous moment in two-dimensional flow with circulation. It is evident that the various vorticity and circulation theorems which are at the foundation of the theory of inviscid potential flow hold equally when the viscosity is not zero. In addition, the theory of viscous potential flow admits approximations to real flows in certain situations. Perhaps if the theory of potential flow had developed after 1850, no one would put $\mu = 0$.

Viscous Potential Flow with Interfaces

$$\mathbf{u} = \nabla \phi, \quad \nabla^2 \phi = 0, \quad \text{Bernoulli equation}$$

Interface conditions



$$\omega = \frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y},$$

Kinematics of moving interfaces

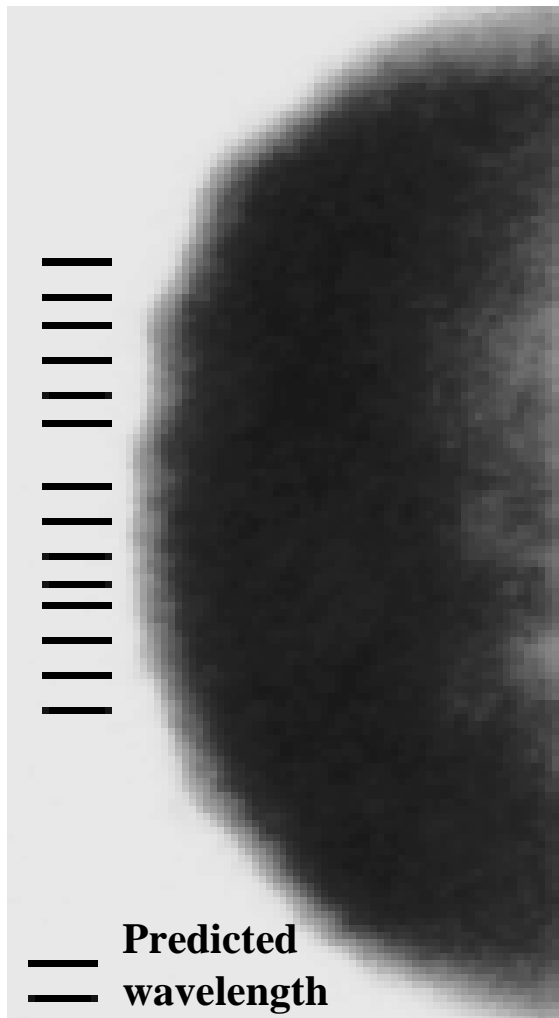
$$-[[p]] + 2\mu \left[\left[\frac{\partial u_n}{\partial n} \right] \right] = \gamma \mathcal{W}_\pi \bullet \mathbf{n},$$

Jump of normal stress
balanced by surface tension

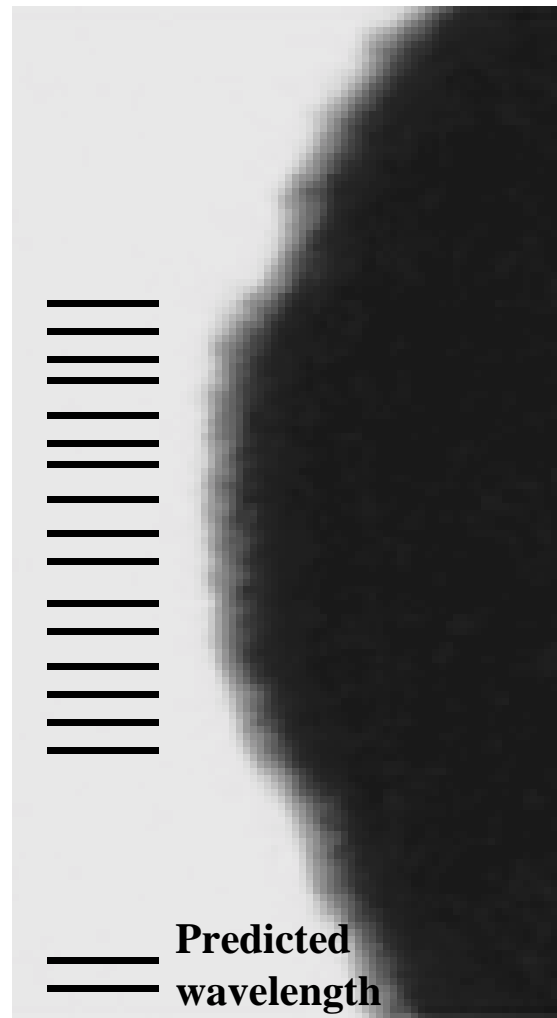
Continuity of tangential component of velocity continuity of the shear stresses **are not enforced**

- Shear stresses are neglected
- Extensional stresses are not neglected

Viscous Potential flow analysis of Rayleigh-Taylor Instability



$Ms = 2$



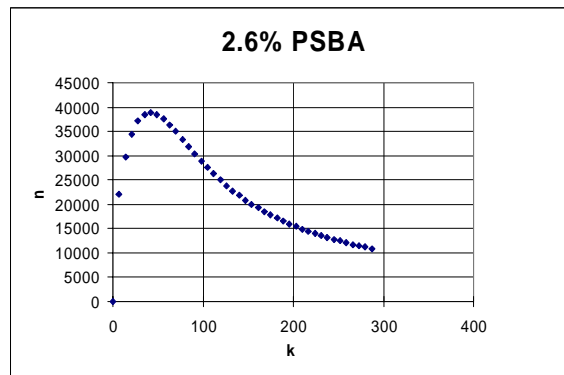
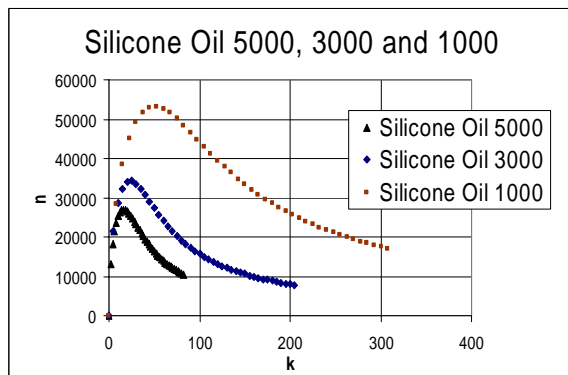
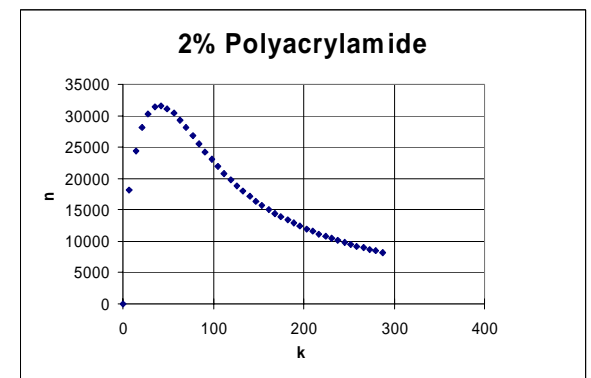
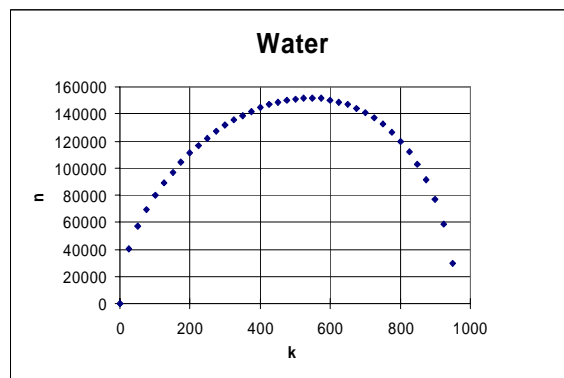
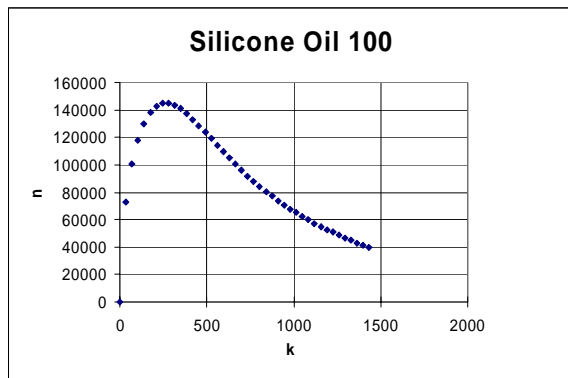
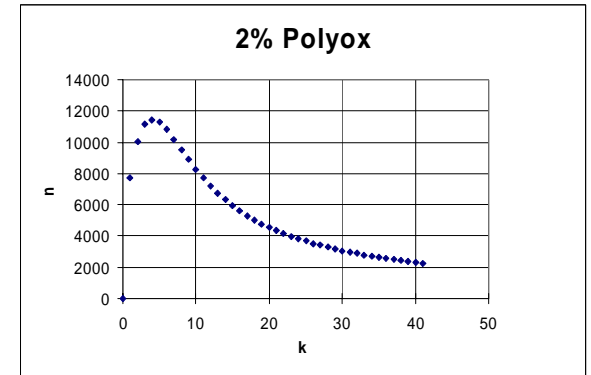
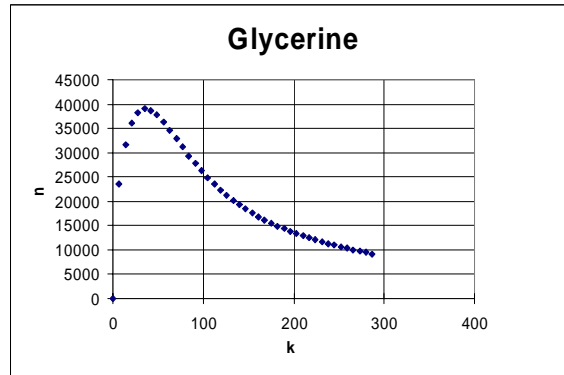
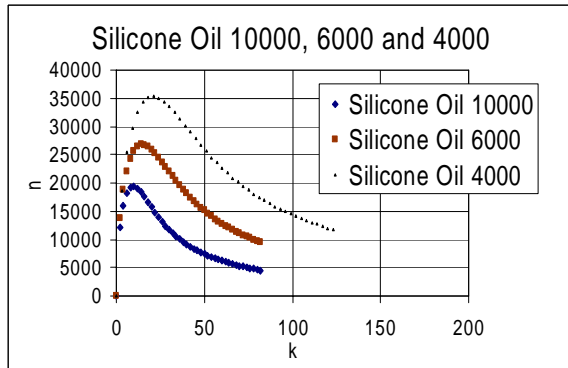
$Ms = 3$

Rayleigh –Taylor Waves in Water.

The tick marks on the photographs locate wave troughs

Joseph, DD, Belanger, J and Beavers, GS 1999. Breakup of a liquid drop suddenly exposed to a high-speed airstream, *Int. J. Multiphase Flow*, **25**, 1263-1303.

n (sec⁻¹) vs. k (cm⁻¹) for Viscous Potential Flow; Shock Mach no. = 3



Values of the wave number, wave length,
and growth rate of the most dangerous wave for the
experimental conditions given in tables 1 and 2.

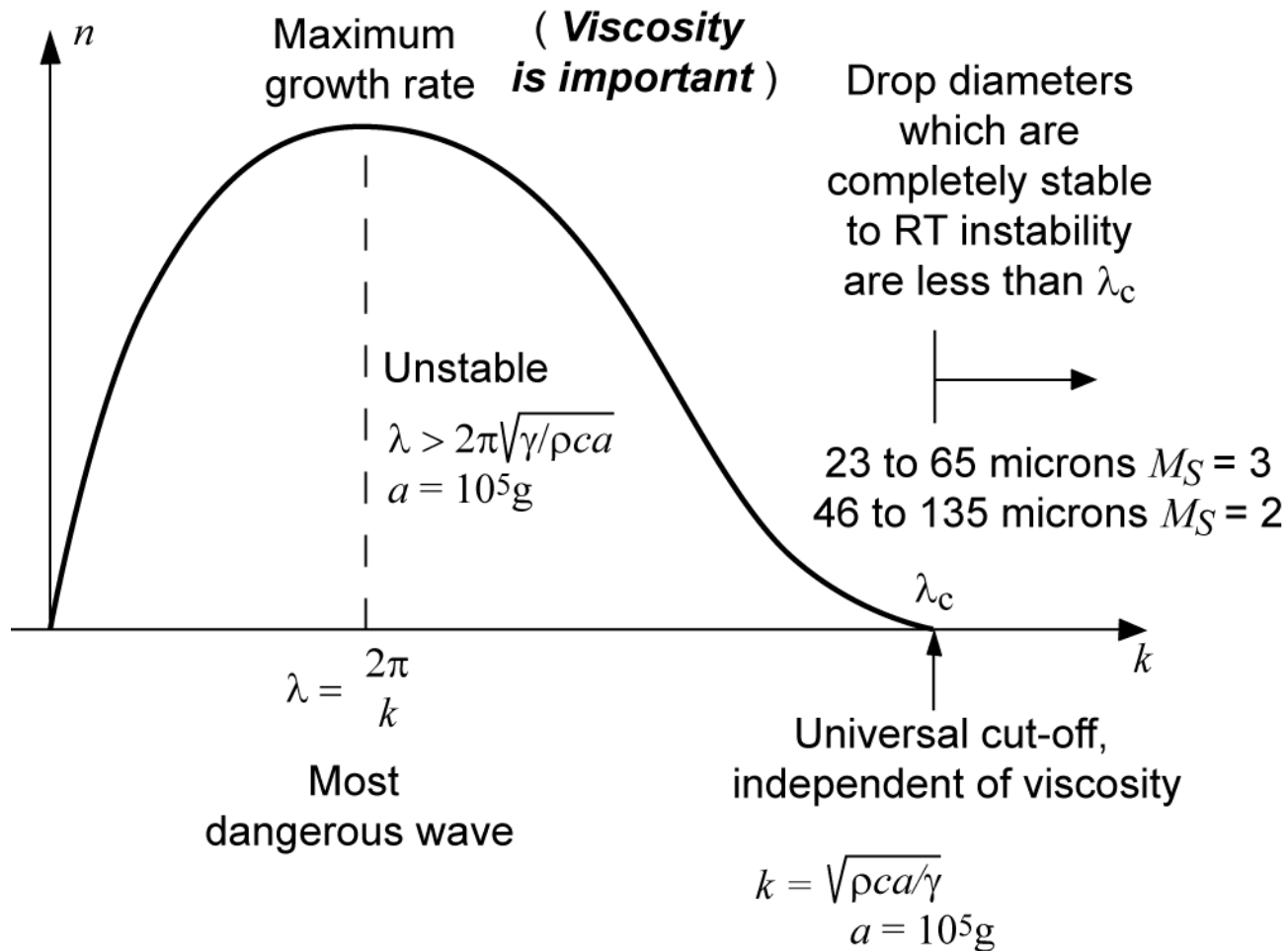
$$M_S = 3$$

Liquid	Viscosity (kg/msec)	Fully Viscous			Viscous Potential		
		n (sec ⁻¹)	k (cm ⁻¹)	λ (mm)	n (sec ⁻¹)	k (cm ⁻¹)	λ (mm)
SO 10000	100	17790	9.5	6.61	19342	9.7	6.48
SO 6000	60	24673	14.45	4.35	26827	14.7	4.27
SO 5000	50	24787	15.86	3.96	26951	16.15	3.89
SO 4000	40	32550	20.3	3.10	35312	20.7	3.04
SO 3000	30	31507	23.08	2.72	34257	23.5	2.67
SO 1000	10	48769	49.68	1.26	53088	50.65	1.24
SO 100	1	132198	253.2	0.25	143699	259	0.24
Glycerine	14.9	38760	41.3	1.52	42141	42.1	1.49
2% PO	350	10492	3.95	15.91	11406	3.95	15.91
2.6% PSBA	11.3	34460	37.8	1.66	37467	38.5	1.63
2% PAA	9.6	28927	39.4	1.59	31451	40.15	1.56
Water	0.01	149632	531.95	0.12	151758	540.8	0.12

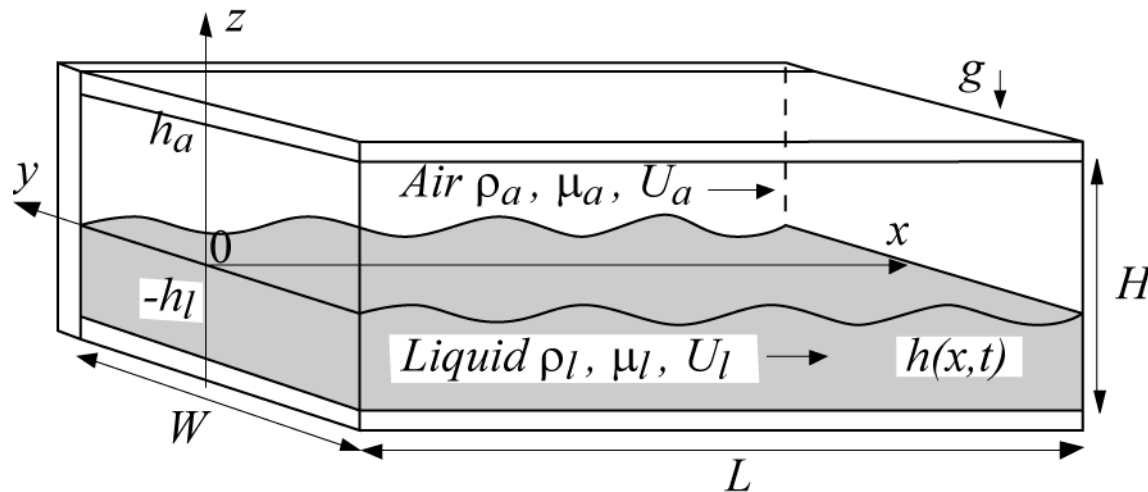
$$M_S = 2$$

Liquid	Viscosity (kg/msec)	Fully Viscous			Viscous Potential		
		n (sec ⁻¹)	k (cm ⁻¹)	λ (mm)	n (sec ⁻¹)	k (cm ⁻¹)	λ (mm)
SO 4000	40	9868	11.2	5.61	10729	11.4	5.51
SO 3000	30	14388	15.6	4.03	15644	15.9	3.95
SO 1000	10	20018	31.8	1.98	21765	32.4	1.94
SO 100	0.1	52726	158	0.40	57304	161.9	0.39
Glycerine	1.49	25046	33.2	1.89	27231	33.8	1.86
2% PO	35	3723	2.35	26.74	4048	2.4	26.18
2.6% PSBA	1.13	16070	25.75	2.44	17472	26.25	2.39
Water	0.001	50971	260.9	0.24	51507	264.2	0.24

Maximum Growth Rate Curve



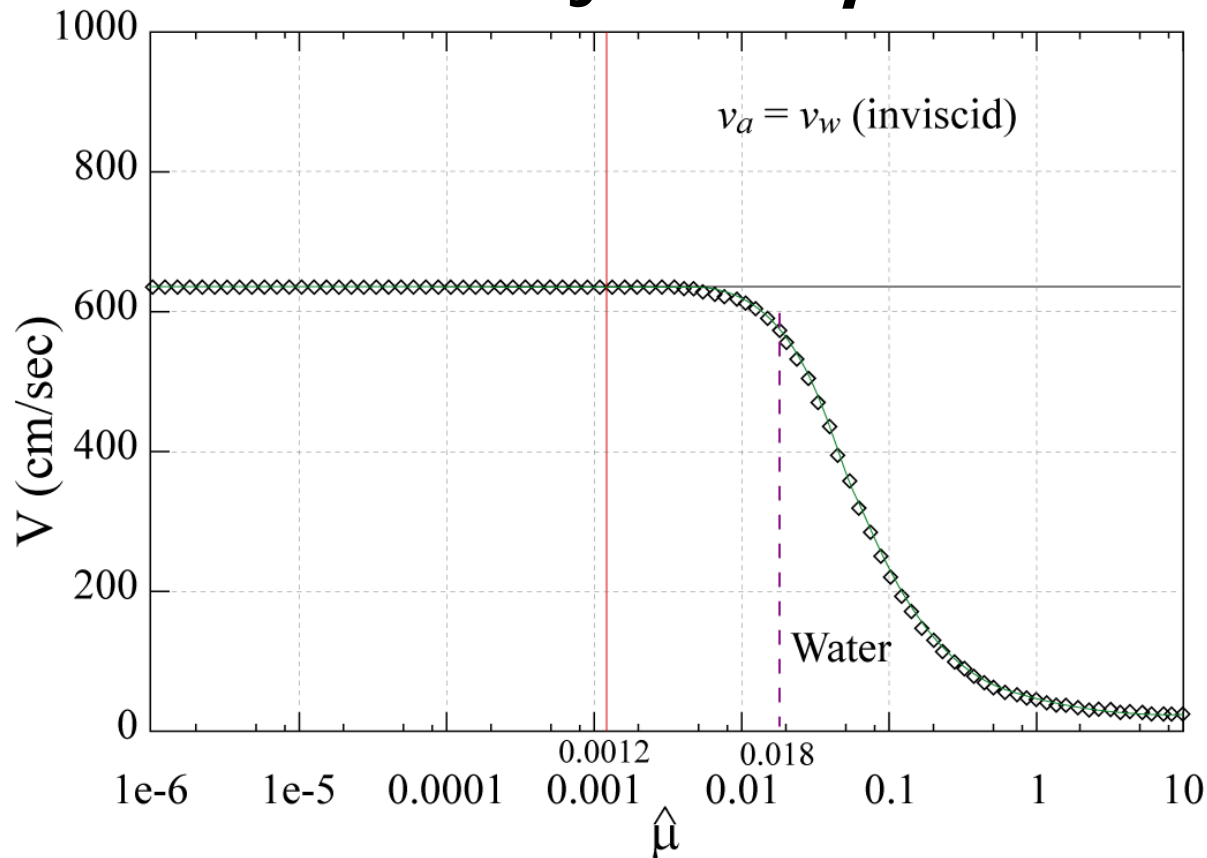
Viscous Potential Analysis of Kelvin-Helmholtz Instability in a channel



Kelvin Helmholtz instability due to a discontinuity of velocity of air above liquid in a rectangular channel. The no-slip condition is not enforced in viscous potential flow so that the two-dimensional solution satisfies the side-wall boundary conditions. There is no exact viscous solution.

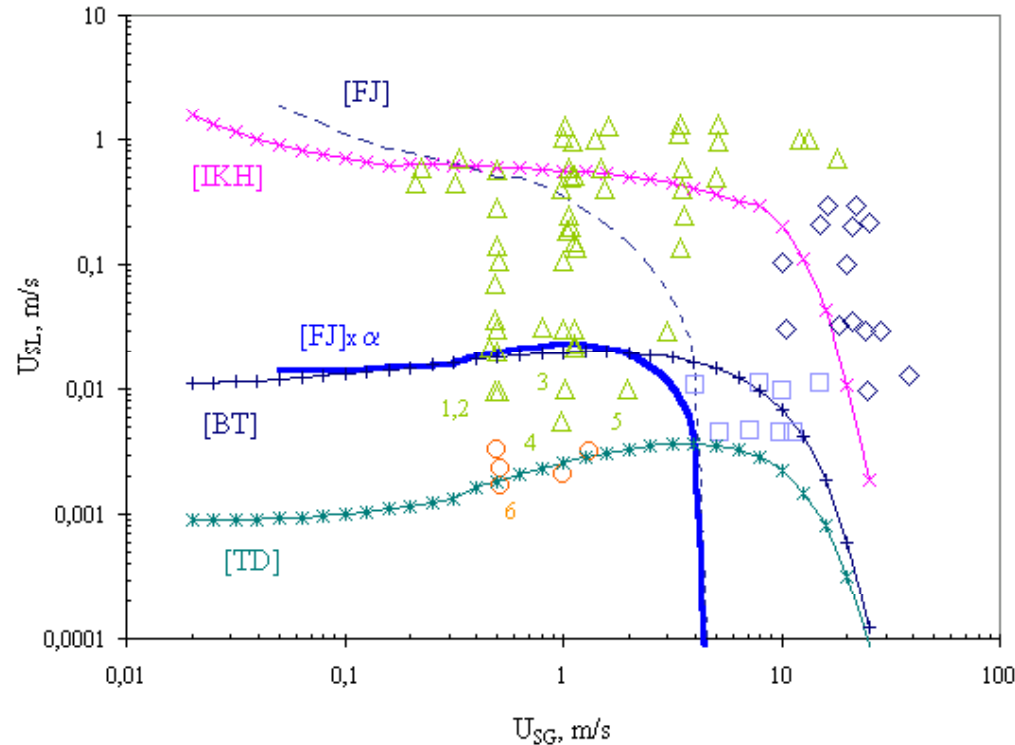
T. Funada and Joseph, DD 2001. Viscous Potential Flow Analysis of Kelvin-Helmholtz Instability in a Channel, *J. Fluid Mech*, **445**, 263-283.

Critical velocity V vs. $\hat{\mu}$ for $a = 0.5$.



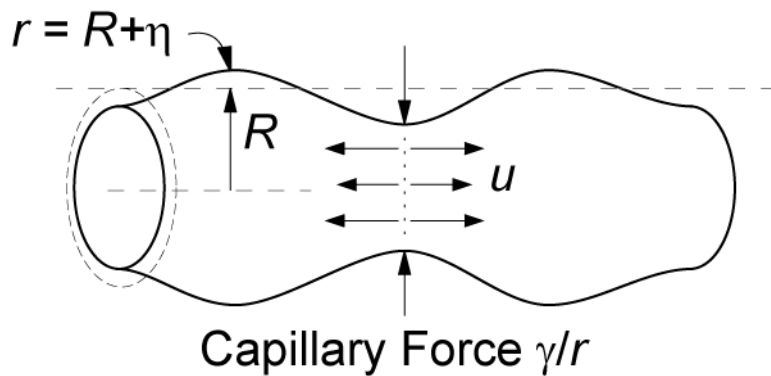
The critical velocity is the minimum value on the neutral curve. The vertical line is $\hat{\mu} = \hat{\rho} = 0.0012$ and the horizontal line at $V = 635.9$ cm/sec is the critical value for inviscid fluids. The vertical dashed line at $\hat{\mu} = 0.018$ is for air and water. Typical values for a high-viscosity liquid are given in table 3 below.

Mandhane flow chart for PDVSA-Intevep data



Mandhane flow chart for PDVSA-Intevep data from 0.508 m I.D. flow loop with air and 0.480 Pa.s lube oil. The identified flow patterns are smooth stratified (SS, open circles), wavy stratified (SW, open squares), slug (SL, open triangles) and annular (AN, open diamonds). Stratified- and non-stratified transition theories after different authors are compared; Taitel and Dukler (1976) [TD]:stars, Barnea and Taitel (1993) [BT]:+, inviscid Kelvin-Helmholtz with [IKH]: \times , Funada and Joseph (2001) [FJ]:broken line, Funada and Joseph multiplied by α (2001) [FJ] $\times\alpha$:heavy line. Notice that the curves [FJ] and [FJ] $\times\alpha$ sharply drop around $U_{SG} \cong 5$ m/s.

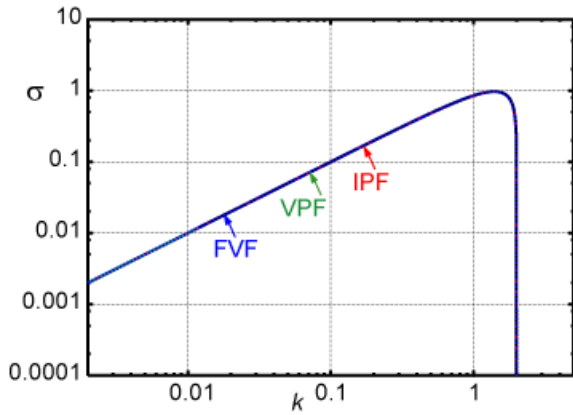
Capillary Instability



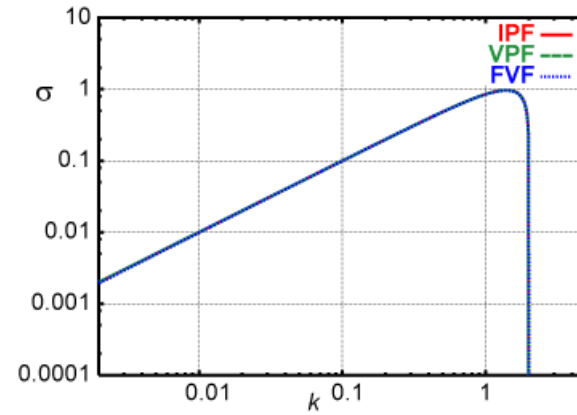
The force $\gamma = r$ forces fluid from the throat, decreasing r leading to collapse.

No. material (fluid l – fluid a)

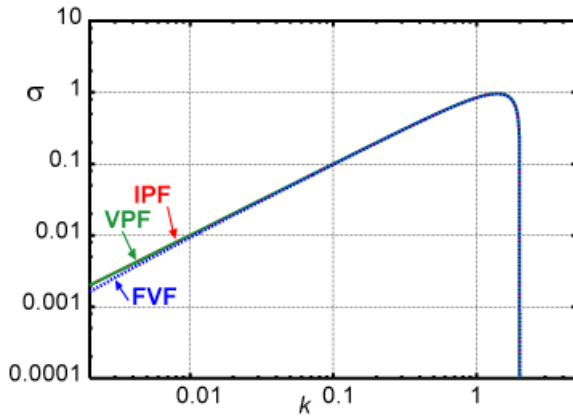
- 1 mercury – air
- 2 mercury – water
- 3 water – air
- 4 benzene – air
- 5 water – benzene
- 6 SO100 – air
- 7 glycerin – mercury
- 8 glycerin – air
- 9 oil – air
- 10 goldensyrup – CC4 and paraffin
- 11 SO10000 – air
- 12 goldensyrup – BB oil
- 13 goldensyrup – Black lubrication oil
- 14 tar pitch mixture – goldensyrup



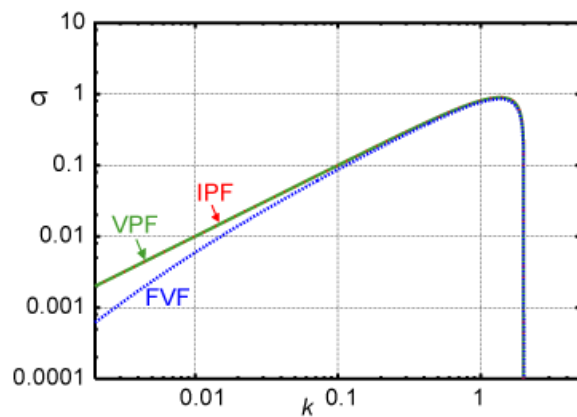
The growth rate σ vs. k for case 1, **mercury in air**



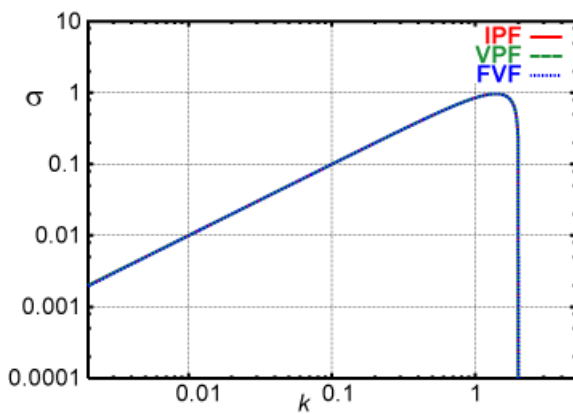
The growth rate σ vs. k for case 4, **benzene in air**



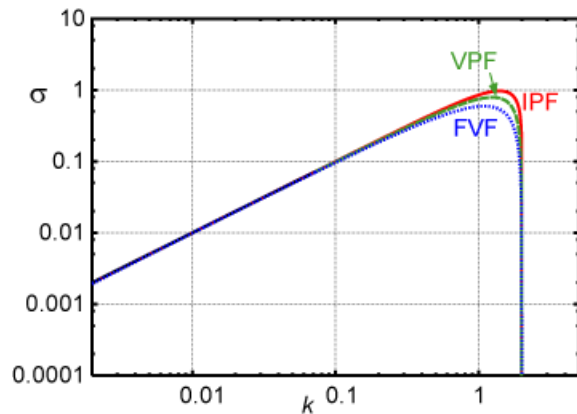
The growth rate σ vs. k for case 2, **mercury in water**



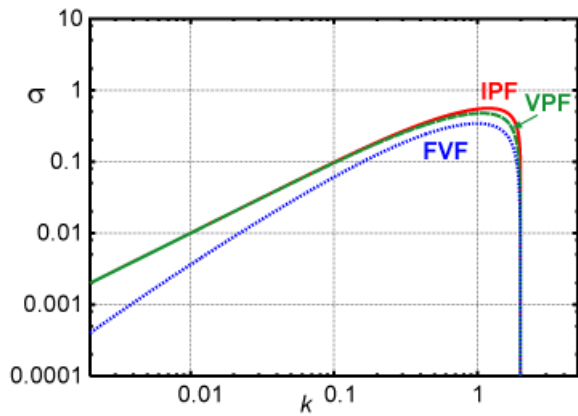
The growth rate σ vs. k for case 5, **water in benzene**



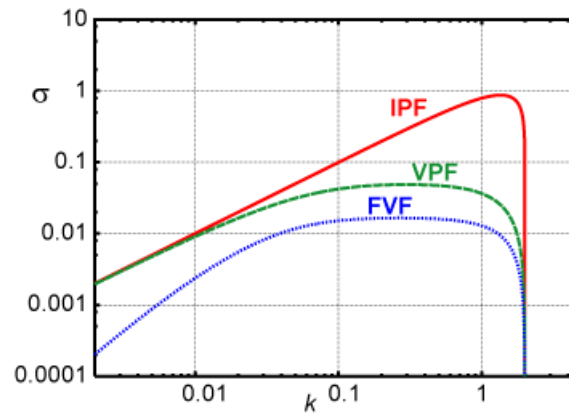
The growth rate σ vs. k for case 3, **water in air**



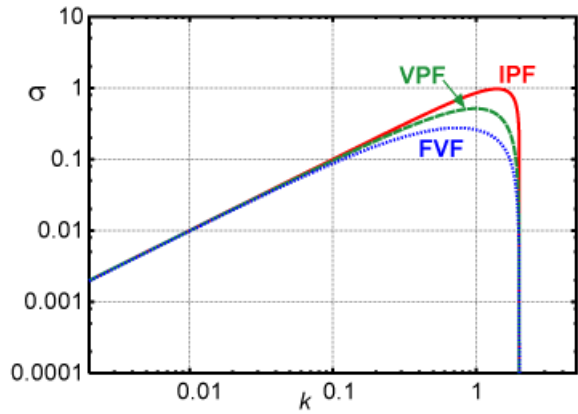
The growth rate σ vs. k for case 6, **SO100 in air**



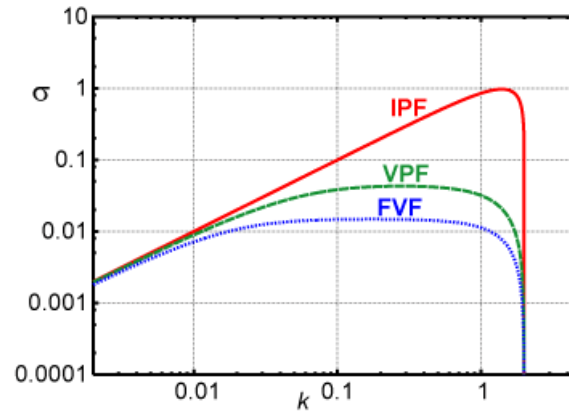
The growth rate σ vs. k for case 7, **glycerin in mercury**



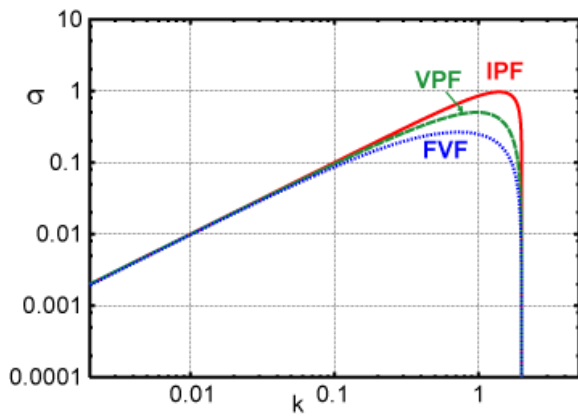
The growth rate σ vs. k for case 10, **goldensyrup in paraffin**



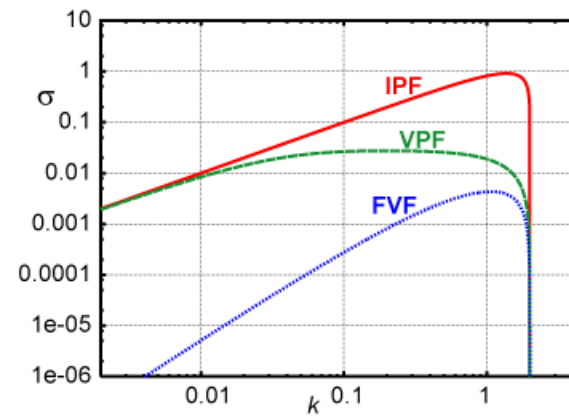
The growth rate σ vs. k for case 8, **glycerin in air**



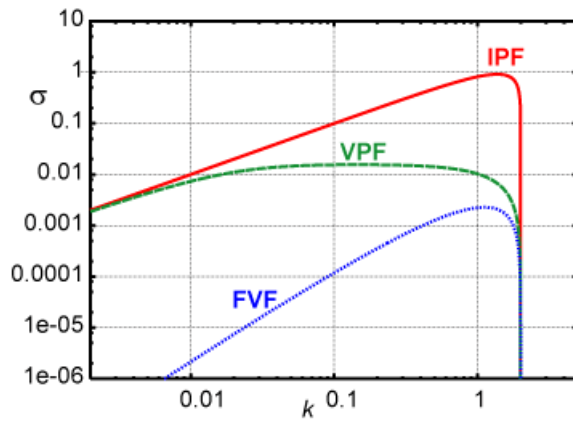
The growth rate σ vs. k for case 11, **SO10000 in air**



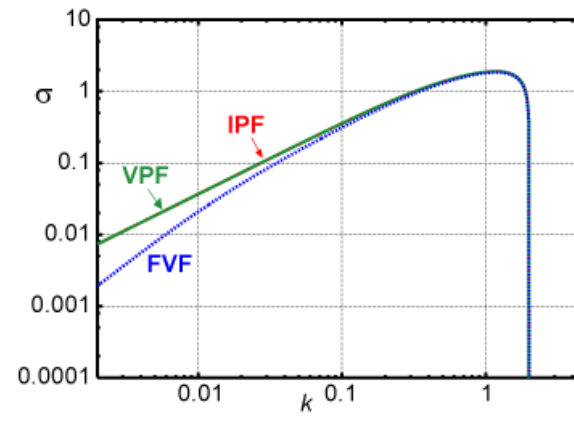
The growth rate σ vs. k for case 9, **oil in air**



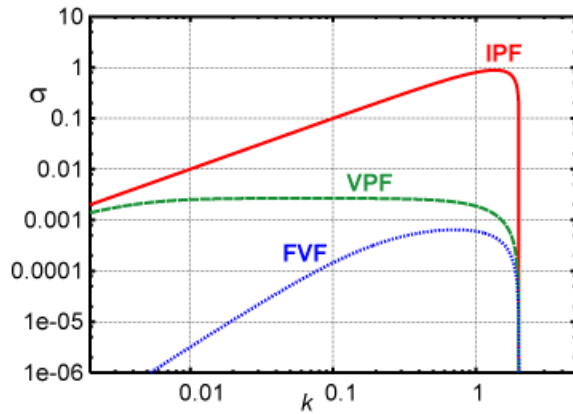
The growth rate σ vs. k for case 12, **goldensyrup in BB oil**



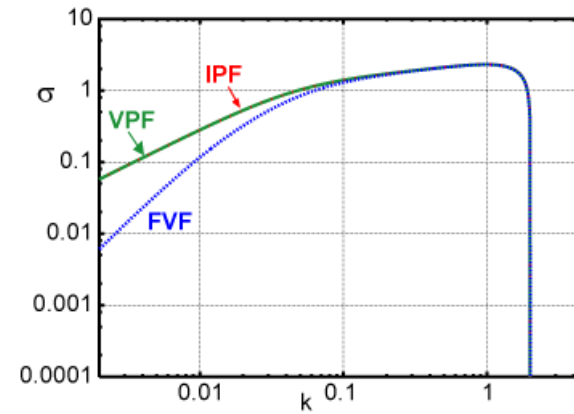
The growth rate σ vs. k for case 13, **goldensyrup in black lubrication oil**



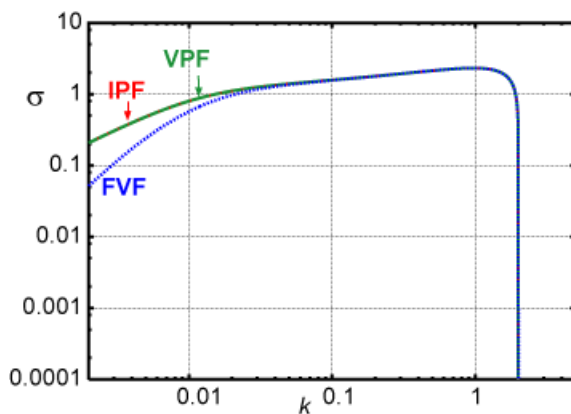
The growth rate σ vs. k for case 2, **(inverse) water in mercury**



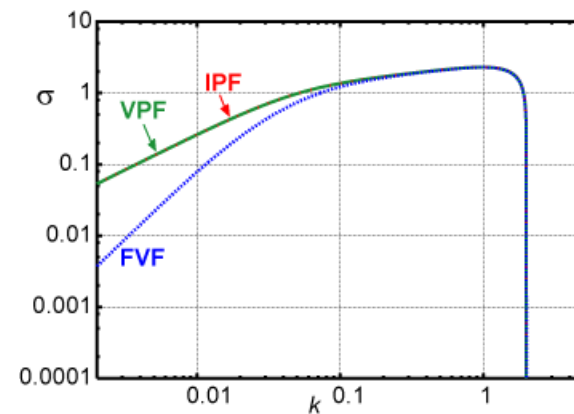
The growth rate σ vs. k for case 14, **tar pitch mixture in goldensyrup**



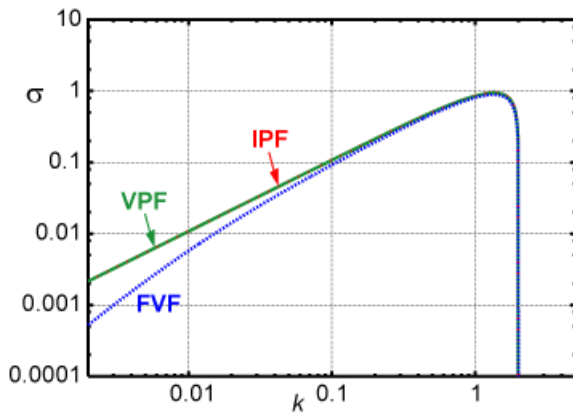
The growth rate σ vs. k for case 3, **(inverse) air in water**



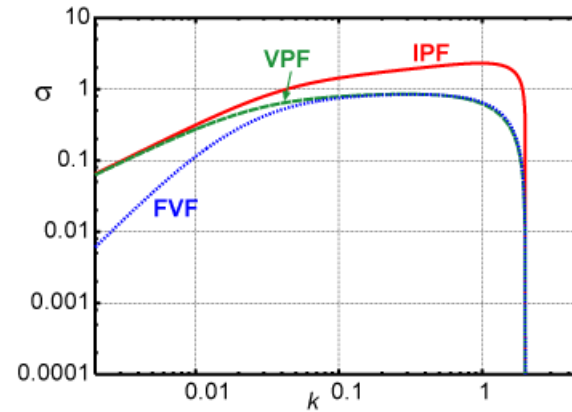
The growth rate σ vs. k for case 1, **(inverse) air in mercury**



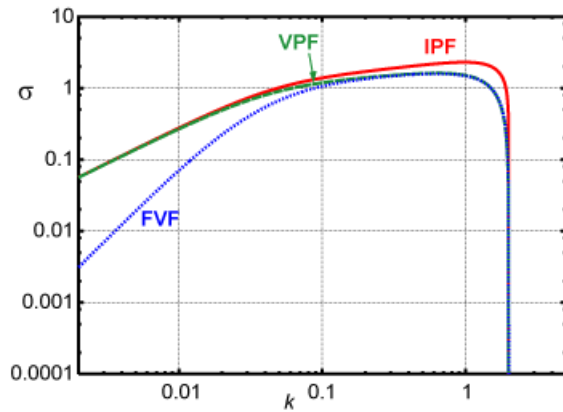
The growth rate σ vs. k for case 4, **(inverse) air in bezine**



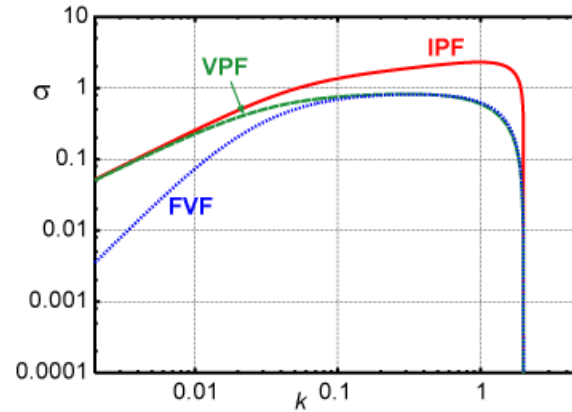
The growth rate σ vs. k for case 5, **(inverse) benzene in water**



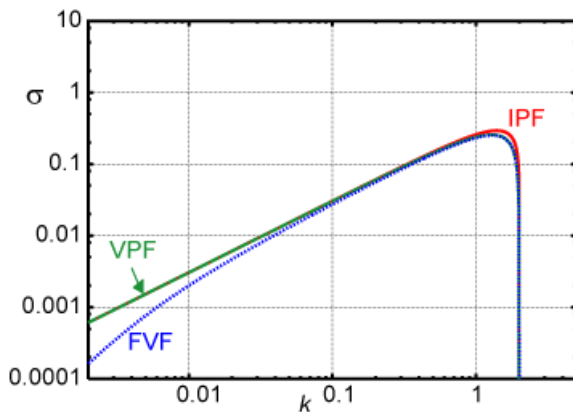
The growth rate σ vs. k for case 8, **(inverse) air in glycerin**



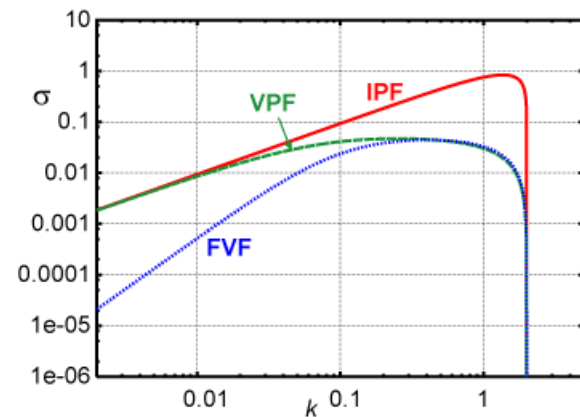
The growth rate σ vs. k for case 6, **(inverse) air in SO100**



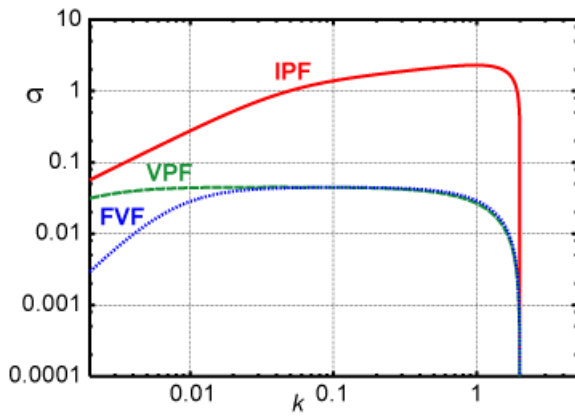
The growth rate σ vs. k for case 9, **(inverse) air in oil**



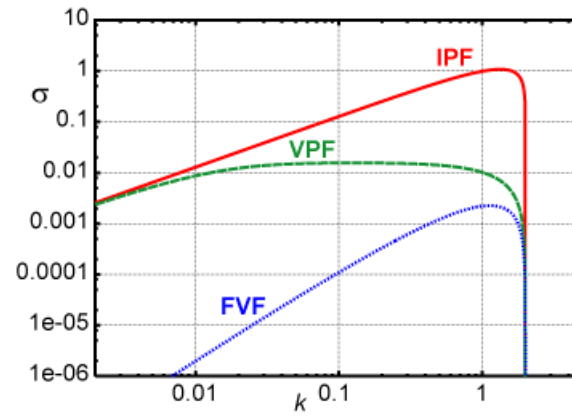
The growth rate σ vs. k for case 7, **(inverse) mercury in glycerin**



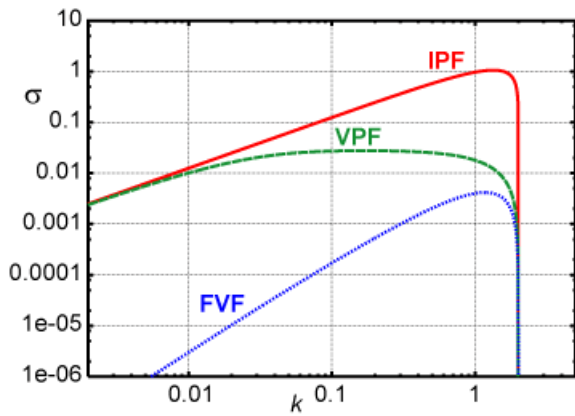
The growth rate σ vs. k for case 10, **(inverse) paraffin in goldensyrup**



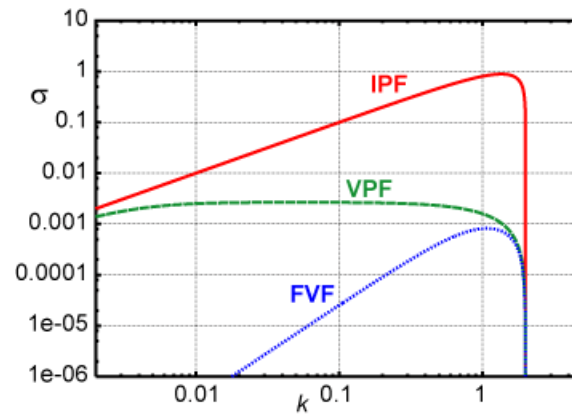
The growth rate σ vs. k for case 11, *(inverse)* air in SO10000



The growth rate σ vs. k for case 13, *(inverse)* black lubrication oil in goldensyrup



The growth rate σ vs. k for case 12, *(inverse)* BB oil in goldensyrup



The growth rate σ vs. k for case 14, *(inverse)* goldensyrup in tar pitch mixture